1. Finding a zero of a function, given an interval in which it changes sign.
2. Finding a local minimum of a function defined on a given interval.
3. Finding the global minimum of a function of one or several variables, given upper bounds on the second derivatives. (This is practical for at most two or three variables.)
4. Finding a local minimum of a function of several variables.

In each case, convergence, the rate of convergence and the effect of rounding errors are analyzed. ALGOL programs that are claimed to be fast and reliable in the presence of roundoff are given along with a substantial amount of numerical results which supports these claims. FORTRAN implementations of the first two algorithms are also given in the Appendix. The book also contains an extensive and up-to-date bibliography relevant to nonlinear optimization.

The reviewer's only complaint about the book is its title which implies that it is about the minimization of nonlinear functions of several variables. Actually, less than one-third of the book deals with this subject. The title, "Algorithms for finding zeros and extrema of functions without calculating derivatives," of an earlier version of the book that appeared as a Stanford University report, gives a more accurate description of the book's contents.

In any case, this book is an excellent reference work for optimizers and root finders who should find the programs in it of great value.

D. G.

33 [**2.45, 10**].—W. W. Boone, F. B. Cannonito & C. R. Lyndon, Editors, *Word Problems. Decision Problems and the Burnside Problem in Group Theory,* North-Holland Publishing Co., Amsterdam and London, 1973, viii + 646 pp., 23 cm. Price \$42.—.

This work is a collection of papers which were presented at a conference at the University of California, Irvine, in September, 1969. A detailed review of all of the 21 papers would require an excessive amount of space; the potential reader will have to consult the "Mathematical Reviews" for an item by item review. However, the book has a unifying theme, namely, the use of algorithmic methods, and this fact is well presented in a very good introduction by the editors who also give proper credit to the pioneering work of Thue and Dehn.

Of the 646 pages of the book, 282 are occupied by a paper on "The existence of infinite Burnside groups" by J. J. Britton. It contains a new proof of the Novikov-Adjan Theorem that, for all sufficiently large odd numbers e, the Burnside groups with exponent e and at least two generators are infinite. The paper is practically self-contained, using little more than the concepts of free group and free product (according to a claim made by the author). Nevertheless, it is probably one of the most difficult mathematical papers ever to appear in print. It is impossible to skim it, and it is very important to know that it has been checked and commented on in detail by J. Mennicke.

Novikov and Adjan had shown that $e \geqq 4381$ is a lower bound (since improved by them) for the Burnside exponent. No such lower bound appears in Britton's paper which, however, establishes a connection between Burnside groups and a generalization of groups investigated by Tartakovskiĭ.

A paper by S. I. Adjan on "Burnside groups of odd exponent and irreducible systems of group identities" in the present volume sketches the author's proof that in a 2-generator group defined by the identities (i.e., relations valid for any pair x, y of group elements)

$$(x^{ke}y^{ke}x^{-ke}y^{-ke})^e = 1 \qquad (e \text{ odd}, e \geqq 4381, k \text{ ranges over all prime numbers})$$

none of the identities can be derived from the others in the system. This implies that there exists a two-generator group with a recursively enumerable set of identities (i.e., a free group of a variety) and with an unsolvable word problem.

The third important paper connected with the Burnside problem is the construction of a nonsolvable group of exponent 5 by S. Bachmuth, H. Y. Mochizuki and D. W. Walkup. The paper is based on the construction of a particular nonnilpotent associative ring and provides also the construction of a Lie ring of characteristic 5 which satisfies the third Engel condition and is not nilpotent. An appendix of the paper contains a FORTRAN program which the authors used to verify some algebraic relations.

These are the contributions to the Burnside problem which fill about half of the volume. The other half contains excellent survey articles and original papers. All of the survey articles concern theories to which the author has contributed heavily himself, but, in some of them, a very large part consists of previously unpublished material. They are surveys only because of the careful outlay of background information and of the references to ramifications. This is true, in particular, of the papers by D. J. Collins: "The word, power, and order problems in finitely presented groups", by C. F. Miller III: "Some connections between Hilbert's 10-th problem and the theory of groups" (written prior to the publication of Matejasevich's famous paper which, however, is briefly considered in an "Added in proof" note) and by D. Tamari: "The associativity problem for monoids and the word problem for semigroups and groups".

The surveys in the usual sense are those by W. Haken: "Connections between topological and group theoretical decision problems", by C. F. Miller III: "Decision problems in algebraic classes of groups" and by P. E. Schupp: "A survey of small cancellation theory". The third one will probably be superseded soon by a forthcoming book by the author and R. C. Lyndon.

The research papers are listed below in alphabetical order:

S. Aanderau: A proof of Higman's imbedding theorem, using Britton extension of groups (18 pp.).

F. S. Cannonito: The algebraic invariance of the word problem in groups (16 pp.).

R. W. Gatterdam: The Higman theorem for primitive-recursive groups—A preliminary report (6 pp.).

S. Lipschutz: On the word problem and T-fourth-groups (10 pp.).

J. McCool and A. Pietrowski: On a conjecture of W. Magnus (4 pp.).

R. McKenzie and R. J. Thompson: An elementary construction of unsolvable word problems in group theory (18 pp.).

T. G. McLaughlin: A non-enumerability theorem for infinite classes of finite structures (4 pp.).

A. W. Mostowski: Uniform algorithms for deciding group theoretic problems (28 pp.).

B. H. Neumann: The isomorphism problem for algebraically closed groups (10 pp.).

H. Schiek: Equations over groups (6 pp.).

L. Wos and G. Robinson: Maximal modules and refutation completeness: semidecision procedures in automatic theorem proving.

It is not possible to comment on these papers in a general review, but it should at least be mentioned that the paper by B. H. Neumann is preparing the ground for the recent Boone-Higman theorem which characterises the intersection of the algebraically closed groups.

An appendix contains fifty research problems. With one exception, the name of the person raising the question is not given. In several cases, it would be very hard to guess since the problem is "in the air".

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34 [4].—HANS J. STETTER, *Analysis of Discretization Methods for Ordinary Differential Equations*, Springer Verlag, New York, 1973, xvi + 338 pp., 24 cm. Price $44.40.

In the eleven years since Henrici's now classical book "Discrete Variable Methods in Ordinary Differential Equations" (J. Wiley & Sons, 1962) appeared, there have been a plethora of new and variants of existing methods in the literature, particularly for the initial value problem which is the major concern of the exhaustive analysis in this excellent book. Professor Stetter places these methods in a solid mathematical framework, and in doing so, extends the existing theory and presents many new results. To quote from the preface: "This text is *not* an introduction to the use of finite-difference methods; rather, it assumes that the reader has a knowledge of the field, preferably including practical experience in the computational solution of differential equations," and from the end of Chapter 1: "In the remainder of this treatise the many practical aspects of the numerical solution of ordinary